This study examines ~~the adequacy of~~ current normative theories of decision-making under uncertainty for their adequacy under circumstances of extreme uncertainty~~, or~~: ~~what may be~~ situations in which information bearing upon the possible consequences of available actions is scanty, marked by gaps, obscure and vague, or on the contrary plentiful and precise but highly contradictory. The latter two, distinguishable information-states will be described as "ambiguous." ~~While~~The study accepts the basic "neo-Bayesian" point of view that reasonable choice ~~decision-making~~ will reflect not only the decision-maker's basic preferences concerning consequences~~,~~ (his values, or payoff function) but his basic judgments (his opinions, ~~or partial~~ degrees of belief, partial beliefs) concerning the likelihood of those consequences; but it raises the question whether the measurement and representation of uncertain opinions in terms of definite numerical probabilities is equally feasible and useful when the evidence underlying partial belief is vague and confusing as when it is less "ambiguous." If not: then how are "vague" opinions to be expressed, and how may consistent actions appropriately be related to them?

Following I.J. Good's usage of "you" as a technical term for "the person ~~who~~ under discussion who is doing the believing, or deciding, or acting, or inferring," the theories considered, or proposed, here are "normative" in the sense that each ~~they purport to be a theory of your own deliberated preferences~~ may be interpreted as an empirical hypothesis on the ~~logical rela~~ pattern of consistency "your" choices would exhibit if you always took time to "stop to think." That is, each purports to be a theory of the structure of your own deliberated preferences. ~~The test of the appropriateness of such a theory for "you"--i.e., for a given decision-maker--is~~ Given your choices among certain pairs of alternative actions, any such theory will define certain choices among other pairs of actions as "inconsistent" with the first set, and certain other choices as "consistent with" or, perhaps, "implied by" the first. ~~The test of the appropriateness of a given theory for a "you"--i.e., for a given decision-maker--~~

How is it reasonable to act when the consequences of your actions are *extremely uncertain*? ~~or "ambiguous" when information bearing upon those consequences is vague or insubstantial, of dubious relevance or contradictory import?~~ Postulates of rational behavior proposed by L.J. Savage in *The Foundations of Statistics* (1954), purporting to apply to decision-making under general conditions of uncertainty, imply a proposition first associated with Daniel Bernoulli (1763): you should act "as if" you assigned numerical "utilities" to the various possible consequences of your actions and, independently, assigned definite, numerical probabilities to all of the events affecting the consequences, then chose that action which "maximized the mathematical expectation of utility." ~~This advice is operationally *meaningful*, for it rules out certain patterns of preferences among gambles (actions whose consequences are uncertain) that could not correspond~~

1. terminal decisions (no further information).

2. "you"

3. conditions of extreme uncertainty: ambiguity; role and measurement of opinions: "probabilities"? Expression of "vagueness"; and its impact.

4. Nature/use of normative theory; validation.

5. For example: Bernoulli vs. mathematical expectation of money.

6. Bernoulli proposition.

7. Deduced as theorem.

8. Don't just act as if; assign probs; but where to get them? Bernoulli dynamited simple methods of probability as price (Bayes, de Finetti).

9. Borel/Ramsey/~~de Fine~~ Savage: two-outcome gambles. Maximize prob of preferred outcome

10. Savage quote. BY ASKING HIMSELF HIS PREFERENCES IN SIMPLE CHOICES, YOU CAN INFER YOUR OWN PROBS, assuming you obey proposition. (or wish to obey it).

11. Measure prob of die; establish numerical probabilities., inequalities.

12. Three-color urn example:

13. What can explain it: not Bernoulli, etc.

14. Related to "vagueness"; Koopman, Good, etc., explain "vagueness." Measure Y^{o},

15. Criteria for "vagueness" (difference of opinions among experts, individual); Hurwicz, Hodges and Lehmann

16. n-color urn example. 9:1; 1:9; 5:5. 1:1.

How is it reasonable to choose among a given set of actions when the consequences of these actions are extremely uncertain? An answer to such a question concerning "reasonable" ~~behavior~~ choice defines a normative theory of ~~behav~~ decision-making. There are various tests of the adequacy of such a theory, corresponding to the various uses for which it might be intended. The theories examined, criticized or supported in this study purport to provide a "logic of choice"; they ~~consist of~~ offer logic-like criteria of consistency in decision-making, with the goals: (a) of enabling a decision-maker to reduce a complex or unfamiliar decision problem into a set of sub-problems in which his intuitive preferences are clear-cut, ~~and from which to~~ and to ~~enable him to~~ "deduce" his ~~preferences in~~ preferred choice in ~~in~~ the former situation from his preferences in the latter; (b) ~~to~~ aiding him to "police" his hasty decisions for choices incompatible with his true preferences and opinions, upon reflection, such "mistakes" being revealed as "inconsistencies" ~~within~~ in terms of the theory. If a theory is to satisfy either of these aims, it must be ~~true~~: (a) that choices among complex alternatives "implied by" your preferences among simpler ones (in terms of the theory) are acceptable to "you" (using I.J. Good's technical usage of "you" as "the person who decides, believes, chooses") upon thorough deliberation; (b) that pairs of choices that are "inconsistent" (in terms of the theory) are not acceptable to you when you reflect upon your preferences and opinions and their implications in the light of the theory. A theory that satisfies these requirements may be interpreted as a theory of your own deliberated preferences. All of the normative propositions examined in this study may be regarded as empirical hypotheses on the way "you" would choose if you always had time, in ~~Ramsey~~ Frank ~~Fram~~ Ramsey's ~~the~~ phrase, to "stop to think." The test of the adequacy of such a hypothesis for "you"--i.e., for a given decision-maker--is ~~that~~ whether or not you find that your deliberated choices, perhaps in contrast to your initial, hasty, ~~choic~~ relatively unconsidered choices, are in good agreement with the implications of the theory. Each theory proposed implies a particular sort of structure to your body of choices; the question for a particular decision-maker is which one provides a pattern of "logical consistency" to which he finds that he wishes to conform, as evidenced by his actual, deliberated preferences.

Normative theories of choice have been proposed by Frank Ramsey (1926), Bruno de Finetti (1937) and L.J. Savage (1952) which have as consequence a proposition first associated with Daniel Bernoulli (1738): "you" should act "as if" you assigned numerical "utilities" to the various possible consequences of your actions and, independently, assigned definite, numerical probabilities to all of the events affecting the consequences of your actions, then chose that action which "maximized the mathematical expectation of utility." I have termed those decision theorists who espouse this proposition in a strict form (with reference to precise, uniquely-~~defined~~ determined probabilities) as normative guide, "neo-Bernoulliansx" (to distinguish them from the broader class of "neo-Bayesians" who assume that "definite" probabilities may be assigned to some but not necessarily all events for purposes of decision-making).

This "as if" proposition impliesxx(1)x that there exists a set of numbers ("von Neumann-Morgenstern utilities") corresponding to the uncertain outcomes of any given risky proposition or "gamble", and a set of numerical probabilities corresponding to the events determining these outcomes, such that the ranking of mathematical expectations of utility computed for various gambles on the basis of these numbers 2ill reflect his actual, deliberated preferences among these gambles.

As stated, this "Bernoulli proposition" may seem neither empirically plausible as a description of all significant, deliberate behavior, nor intuitivelycompelling as a general standard of reasonablness or desriable consistency in choice. What has made it, in recent years, the focus of widespread interest, controversy, analysis and advocacy has been its derivation as a theorem from various sets of axiomatic propositions ~~that~~ which have themselves, in considerable degree, both these appealing properties. This approach, begun by Frank Ramsey in 1926 and developed most fully by L. J. Savage in the last decade, has demonstrated that any pattern of choice contradicting the "existence ~~theorem~~ assertion" above must also violate certain proposed canons of reasonable behavior that are, undeniably, far more persuasive as normative criteria than is the "Bernoulli proposition" itself.

It is possible to test indirectly the adequacy of any set of postulates leading to the "Bernoulli proposition" as a normative theory for a given decision-maker, by testing that proposition directly in terms of his pattern of deliberated choices. An assertion that a given decision-maker or a given body of choices is "reasonable" in the sense of this ~~text~~ proposition could be refuted by specifiable data on choices; for there are patterns of preference among gambles that could not correspond to ~~an~~ a ranking of mathematical expectations of utility based upon any hypothetical probabilities or utilities, however determined.

On the other hand, there are patterns of choice among certain sorts of ~~ga~~ simple gambles that ~~would~~ would be consistent with the proposition on the basis of one and only one set of numerical probabilities. Thus, a decision-maker who wishes his choices to be consistent with the "Bernoulli proposition" or with some set of postulates leading to it (or who has found, by reflection upon past choices, that his deliberated, "careful" decision-making is invariably consistent with such criteria), can "observe" his "definite" preferences with respect to such gambles, "measure" his own probabilities for the events in question, and use these probability numbers in calculating expected utilities, hence preferences, for complex gambles involving these same events. As the author of a "neo-Bernoullian" textbook has put it, "the author believes...that when the consequences of various possible courses of action depend on some unpredictable event, the practical way of choosing the "best" act is to assign values to consequences and probabilities to events and then to select the act with the highest expected value."

(S-v)

Since the postulates of reasonable decision-making advanced by Savage in The Foundations of Statistics (1954) lead to the "Bernoulli proposition," a decision-maker who accepts them as normative criteria can use that proposition in his own decision-making: he can (a) construct (if necessary) hypothetical gambles of

If utility numbers appropriate to the "Bernoulli proposition" -- currently known as "von Neumann-Morgenstern utilities"--are known or assumed for the given decision-maker, then unique probabilities ~~can be inferred~~ consistent with the "Bernoulli proposition" (if ~~any~~ his choices are consistent with thatproposition at all; i.e., if any exist) can be inferred from the ~~price~~ maximum ~~price~~ offer (in utility terms) he is willing to make for a particular contingency, i.e., for a prize of unit utility in case the event actually obtains. In this context, probability emerges as a price (of a contingency offering unit utility contingent upon the event's obtaining, or occurring); equivalently, it can be defined in terms of the least
(in utilities)
favorable odds/he will accept in betting on the event's obtaining. However, such
assumed money values
definitions--proposed by Thomas Bayes, Bruno de Finetti, and discussed by Ramsey and Savage--demand prior measurement of utilities to be applicable; if we accept
MENTION THIS
Daniel Bernoulli's counterexamples to the principle of the maximization of expected money, then we cannot assume money values to be appropriate for this calculation of prices (i.e., we cannot assume utility to be linear on money; we cannot predict, or prescribe as reasonable, prices to be paid for gambles based on prior observations involving different amounts of money). (ISN'T THIS WHOLE APPROACH BASED ON A RATIONALISATION OF "DEFINITE" PROBABILITIES THAT REALLY "RATIONALIZES" THE USE OF EXPECTED MONEY VALUES?)

The Ramsey/Savage approach permits not only the comparison of the probabilities of any two events, in terms of inequalities, but in special cases their numerical measurement, without assuming either prior measurement of the utility function or utility as a linear function of money. The only choices that need be considered are choices among gambles all offering the same two outcomes, one definitely preferred to the other, these outcomes being associated with various different subsets of events. In these circumstances the principlesof maximizing the mathematical epxecation of utility reduces to the principle of maximizing the probability of the preferred outcome. Thus, if we offer to stake a given prize alternatively upon define: staking, or offering, a prize

event A or event B~~, your~~ (i.e., ~~to offer to pay~~ if we offer to pay you a given amount in case A obtains, giving you nothing if A does not obtain; or alternatively, ~~in ca~~ to pay you a given amount in ~~cas~~ case B obtains, nothing if B does not obtain), your definite preference for, say, staking the prize on A would be compatible with the "Bernoulli proposition" only ~~if~~ on the basis of a probability for A greater than that for B.

"If I offer to pay you $10,00 if any one horse of your choosing wins a given race, your decision tells me operationally which you consider to be the most probable winner."

If we assume you obey the "Bernoulli proposition," at least in your careful deliberated choices, then by knowing which outcome you prefer and which events you prefer to stake it on, we can infer which events you regard as "more probable" than others. Moreover, if you are indifferent between staking it on two or more given events, then only equal probabilities would be at once compatible with this indifference and with the Bernoulli proposition. Finally, if you are indifferent between staking the prize on any one of n mutually exclusive events given that (you are certain that) one of them will obtain, then the only numbers with the characteristics of a probqbility measure compatible with these choices and with the Bernoulli proposition are (1/n,......1/n). I.e., these are the only numbers compatible with your choices and with the Bernoulli proposition, if any are; hence, on the assumption that you "obey" the Bernoulli proposition, or propositions implying it, we have "measured" your probabilities, which can thenceforth be used to calculate, or predict, or prescribe your further choices involving these events. Moreover, by ~~comparing your~~ offering you prizes staked on other events or, alternatively, on one of these events, we can meaningfully establish numerical inequalities for the probabilities of these other events; i.e., deduce limits within which a given numerical probability must lie; if the postulates are all obeyed, these limits can be made indefinitely narrow. Thus, the convenience of the Savage/Bernoulli theory, in describing/calculating/predicting/prescribing choices is clear: ~~if~~ to the extent that it is acceptable to a given decision-maker.

This study focusses upon the question of decision-making under conditions of extreme uncertainty : DEFINE (p. 1, etc.) Ramsey etc. theory applies to all conditions of uncertainty: test if first.

EXAMPLE of above: a die, measure probability equal to probability = 1/6, etc.

Three-color urn example: show how Bernoulli proposition is meaningful (implies choices) and would be applied; suppose offer is between III and IV. Ask yourself: Yellow or Black: Red=1/3. Then: I=II, III=IV, same offers for both.

But suppose you prefer IV to III; do you want to change earlier answers? No; well anyway, then, you prefer II to I . No. Then you're indifferent between I and II. No.

or with probs

Inconsistent with Bernoulli proposition. Hence with Savage postulate: 2.

For such people, this doesn't provideadequate normative theory. What does?

Try other criteria WHICH REQUIRE ONLY THE DATA PROVIDED SO FAR: ON THE VALUES OF THE PAYOFFS: minimax regret, Hurwicz, minimax, Laplace: No.

Other criteria exist, but require more data (for decision-maker): intuitive judgments of probability. Keynes/Koopman/Good, partial ordering

Use same n-scale of propositions.

Define Y^o; y^o. Simplest rule: Type II minimax (maximize lower probability). But more complex rules explain more behavior: maximax element (n-color urn example); increased sample effect, or more information, "confidence." Includes all earlier decision rules: provides role of "confidence" (Knight) and boldness/prudence. Can explain randomization (though this is cited as criticism).

Chipman, Raiffa, Becker/Brownson, my seminars, me.

Subject: reasonable choice under conditions of extreme uncertainty: normative theory: logic of choice, purposes, theory of your deliberated preferences: police actions for consistency, deduce new ones from old. Test: your own preferences, over time

Bernoulli proposition: meaningful; leads to Schlaifer advice; but violated in our example (three-urn, n-color urn);

What can explain: not minimax, minimax regret, Hurwicz, Shackle, Laplace, Bernoulli; but restricted Hurwicz, restricted Bayes/Hurwicz

/first: explain Koopman/Good intuitive probabilities